

# Multivariate Analysis for TOF-SIMS 

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## Outline

- Thinking Multivariate
- General Principles
- Data Sets
- Pattern Recognition with Principal Components Analysis
- Preprocessing
- Supervised Pattern Recognition: Classification
- Analysis of Multivariate Images
- Self Modeling Mixture Analysis, aka Curve Resolution
- Clustering
- Conclusions


## Definition of Chemometrics

Chemometrics is the chemical discipline that uses mathematical and statistical methods to

1) relate measurements made on a chemical system to the state of the system 2) design or select optimal measurement procedures and experiments.

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## Multivariate Analysis

Multivariate Statistical Analysis is concerned with data that consists of multiple measurements on a number of individuals, objects, or data samples. The measurement and analysis of dependence between variables is fundamental to multivariate analysis.

# Information Hierarchy 



## Motivation: Which Point is Most Unique?



X2 with 95\% Confidence Limits


## Plot X2 versus X1



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## Principal Component Scores


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Monitor Single $T^{2}$ Chart


## General Principles

- Balance
- "Let the data speak for itself" - Bruce Kowalski
- "Don't estimate what you already know" - John MacGregor
- Easier to fit data than predict it
- Remember the parsimony principle
- Validate models on independent test sets
- What you do before PCA, PLS etc. is critical
- Experimental design, sample pedigree
- Preprocessing to eliminate unwanted variance


## Example Data Set 1

- Tyrosine-derived polyarylates
- From polymerization of diacids and diphenols
- Backbone length varied (X)
- Pendent (side) chain length varied (Y)



## Example Data Set 2

- Multilayer drug beadcontrolled release delivery system
- TOF-SIMS taken of cross section of bead
- Evaluate integrity of layers, distribution of consituents


Thanks to Anna Belu!
A.M. Belu et. al., "TOF-SIMS Characterization and Imaging of Controlled-Release Drug Delivery Systems, Anal. Chem., 72(22), pps 5625-5638, 2000

## Principal Components Analysis



## PCA

- Geometry for 2 variables


Variable 1


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## City Streets Analogy

Puget Sound

## PCA Math 1 of 2

For a data matrix $\mathbf{X}$ with $m$ samples and $n$ variables (generally assumed to be mean centered and properly scaled), the PCA decomposition is:

$$
\mathbf{X}=\mathbf{t}_{1} \mathbf{p}_{1}^{\mathrm{T}}+\mathbf{t}_{2} \mathbf{p}_{2}^{\mathrm{T}}+\ldots+\mathbf{t}_{\mathrm{k}} \mathbf{p}_{\mathrm{k}}{ }^{\mathrm{T}}+\ldots+\mathbf{t}_{\mathrm{q}} \mathbf{p}_{\mathrm{q}}^{\mathrm{T}}
$$

Where $\mathrm{q} \leq \min \{\mathrm{m}, \mathrm{n}\}$, and the $\mathbf{t}_{\mathbf{i}} \mathbf{p}_{\mathrm{i}}{ }^{\mathrm{T}}$ pairs are ordered by the amount of variance captured.

Generally, the model is truncated, leaving some small amount of variance in a residual matrix:

$$
\mathbf{X}=\mathbf{t}_{1} \mathbf{p}_{1}{ }^{\mathrm{T}}+\mathbf{t}_{2} \mathbf{p}_{2}^{\mathrm{T}}+\ldots+\mathbf{t}_{\mathrm{k}} \mathbf{p}_{\mathrm{k}}{ }^{\mathrm{T}}+\mathbf{E}=\mathbf{T}_{\mathrm{k}} \mathbf{P}_{\mathrm{k}}^{\mathrm{T}}+\mathbf{E}
$$

## PCA Math 2 of 2



The $\mathbf{p}_{i}$ are eigenvectors of the covariance matrix of $\mathbf{X}$

$$
\begin{aligned}
& \operatorname{cov}(\mathbf{X})=\frac{\mathbf{X}^{\mathrm{T}} \mathbf{X}}{\mathrm{~m}-1} \\
& \operatorname{cov}(\mathbf{X}) \mathbf{p}_{\mathrm{i}}=\lambda_{\mathrm{i}} \mathbf{p}_{\mathrm{i}}
\end{aligned}
$$

and $\lambda_{i}$ are eigenvalues.
Amount of variance captured by $\mathbf{t}_{\mathbf{i}} \mathbf{p}_{\mathrm{i}}{ }^{\mathrm{T}}$ proportional to $\lambda_{\mathrm{i}}$.

## Properties of PCA

- $\mathbf{t}_{\mathbf{i}}, \mathbf{p}_{\mathbf{i}}$ ordered by amount of variance captured
- $\mathbf{t}_{\mathbf{i}}$ or scores form an orthogonal set $\mathbf{T}_{k}$ which describe relationship between samples
- $\mathbf{p}_{\mathrm{i}}$ or loadings form an orthonormal set $\mathbf{P}_{\mathrm{k}}$ which describe relationship between variables
- scores and loadings plots are interpreted in pairs
- e.g. plot $\mathbf{t}_{\mathbf{i}}$ vs sample number and $\mathbf{p}_{i}$ vs variable number
- it is useful to plot $\mathbf{t}_{i+1}$ vs. $\mathbf{t}_{\mathrm{i}}$ and $\mathbf{p}_{\mathrm{i}+1}$ vs. $\mathbf{p}_{\mathrm{i}}$


## Variable Loadings, $p_{i}$



## Sample Scores, $t_{i}$



## Arylate Data



## PCA of Mean-centered Arylate

Percent Variance Captured by PCA Model



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Log-decay Scaling
Raw Data



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## PCA with Log-decay, MC

Percent Variance Captured by PCA Model


Can we do better? Normalize?


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## Log-decay, Normalize, Mean-Center

Percent Variance Captured by PCA Model


## How Does it Work on the Test Set?



Check residuals!


## Geometry of Q and T²



## Supervised Pattern Recognition

- A single PCA model worked fine to visually classify arylates for backbone length
- PCA models could be built of each class (SIMCA)
- Fairly obvious this would work well



## Apply SIMCA to Arylate for Sidechain?

- Doesn't work because major variation in spectra (with this scaling) due to backbone, not side chain
- Try discriminant analysis instead


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## Partial Least Squares Discriminant Analysis (PLS-DA)

- Use PLS regression to determine axis to project data on that discriminates between classes
- choose axis so individual distributions are narrow
- choose axis so centers of distributions are far apart
- PLS is factor-based model of data therefore more stable with high collinearity.
- Will automatically attempt to identify directions of interest!


## PLS-DA for Sidechain Length

samplesicores fiot or aryare_cai,c a aryare_cest,
Calibration and test samples shown


## Image PCA

- SIMS images contain complete spectra for each pixel
- Use PCA to condense information from all channels down
- Use "scores" instead of single channels



## Perform PCA on Unfolded Data



## Refold Results from PCA



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Total Ion Image of Bead


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## Scores on First PC



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## Scores on Second PC



## Scores and Loads on Second vs. First PC



## Problem: Not Much Contrast!



Contrast Enhanced Scores on PC 1

Scores on PC\# 1


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## Histogram of PC1 Scores Afer Contrast Enhancement



## Contrast Enhanced Scores on PC 2

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## Contrast Enhanced Scores on PC 3



## Contrast Enhanced False Color Image



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## Maximal Autocorrelation Factors (MAF)

- Regular image PCA does not take any spatial correlations into account, just captures variance
- MAF finds factors which capture large amounts of variance and produce correlated scores in the image plane
- Result is that features with large spatial correlations move up in model


## PVA Image Data PCA vs. MAF Score Images



## MCR Objective

- Decompose a data matrix into chemically meaningful factors
- pure analyte spectra
- pure analyte concentrations
- Easy to interpret
- provides chemically / physically meaningful information
- caveats:
- rotational and multiplicative ambiguity
- use of constraints


## MCR

- Based on the classical least squares (CLS) model, attempt to estimate $\mathbf{C}$ and $\mathbf{S}$ given $\mathbf{X}$ :
$\mathbf{X}=\mathbf{C S}^{T}+\mathbf{E}$
where
$\mathbf{X}$ is a $M \mathrm{x} N$ matrix of measured responses,
$\mathbf{C}$ is a $M \mathrm{x} K$ matrix of pure analyte contributions, $\mathbf{S}$ is a $N \mathrm{x} K$ matrix of pure analyte spectra, and $\mathbf{E}$ is a $M \mathrm{x} N$ matrix of residuals.


## Alternating Least Squares

- How can we improve estimates of $\mathbf{S}$ and $\mathbf{C}$ ?
- Given initial guess $\mathbf{S}_{0}$ (or $\mathbf{C}_{0}$ )...

$$
\begin{aligned}
& \mathbf{C}_{i}=\mathbf{X} \mathbf{S}_{i-1}\left(\mathbf{S}_{i-1}{ }^{\mathrm{T}} \mathbf{S}_{i-l}\right)^{-1} \\
& \mathbf{S}_{i}=\left(\mathbf{C}_{i}{ }^{\mathrm{T}} \mathbf{C}_{i}\right)^{-1} \mathbf{C}_{i}^{\mathrm{T}} \mathbf{X}
\end{aligned}
$$

- Iterate until convergence (ALS)
- Usually constrained such that $\mathbf{C}>0$ and $\mathbf{S}>0$
- and each $\mathbf{s}_{k}{ }^{T} \mathbf{s}_{k}=1$


## Initial Estimate

- Try to find "extreme" samples/pixels
- Or look for "extreme" variables



## MCR (ALS) on TOF-SIMS Image

- Non-negative constraints on both C and S
- Initialize with pure samples (i.e. pixels)
- Recover 6 interpretable spectra and concentration profiles
- Showing Score Images - image was unfolded with each pixel as a separate sample then the scores are re-folded to form images


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## RGB "Chemical" Image

| Red: Surelease (bead coating) |
| :--- |
| Green: Na |
| Blue: Prednisolone (drug) |

only 3 of 6 factors extracted are shown


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## k-Means Agglomerative Clustering

- Samples are paired with another sample or a cluster one-at-atime
- Position of each cluster is mean of all samples in cluster.
- Recalculation of distance can take a long time with lots of samples



## KNN vs. K-Means

Two clusters are grouped together when...

KNN
...two of their members are the closest of all dissimilar samples


Note: these rules apply even when one of the "groups" is a single sample in a group of its own.

K-Means
...the cluster means are the closest of all cluster means


## k-Means Partitional Clustering

- Choose k samples as cluster "targets"
- random selection of samples
- "pure samples": choose samples on outside of data (furthest from all other samples)
- Classify all samples into one of those $k$ clusters.
- Calculate mean of each cluster's samples
- Repeat classification and cluster means until no samples are re-classed after mean recalculation.
- Much faster, but dependent on initial guess of samples


## Avicel by k-means Clustering

False-color MCR Results


Pure Pixel Clusters

(3 clusters)

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# Why Multivariate and Factor Based Methods? 

- Noise filtering
- Selectivity enhancement
- Interpretation
- It's a multivariate world!

Eigenvector offers a range of prepackaged and custom software products. Both as add-ond to MATLAB and as stand-alone software.

PLS_Toolbox 4.0
Solo 4.0
Model_Exporter 1.0
MIA_Toolbox 1.0
EMSC_Toolbox 1.0

## Resources

## - Books

- Chemometrics, M.A. Sharaf, D.L. Illman and B.R. Kowalski, Wiley-Interscience (1986) ISBN 0-471-83106-9
- Multivariate Analysis, K.V. Mardia, J.I. Kent and J.M. Bibby, Academic Press, (1979) ISBN 0-12-471252-2
- Multivariate Calibration, H. Martens and T. Næs, John Wiley \& Sons Ltd. (1989) ISBN 0-471-90979-3
- Chemometrics: a textbook, D.L. Massart et al., Elsevier (1988) ISBN 0-444-42660-4
- Chemometrics: A Practical Guide, K.R. Beebe, R.J. Pell, M.B. Seasholtz, Wiley (1998) ISBN 0-471-12451-6
- Multivariate Data Analysis In Practice, Kim H. Esbensen, CAMO ASA (2000), ISBN 82-993330-2-4
- A user-friendly guide to Multivariate Calibration and Classification, T. Næs, T. Isaksson, T. Fearn, T. Davies, NIR Publications(2002), ISBN 0-9528666-2-5
- Multivariate Image Analysis, Paul Geladi and Hans Grahn, Wiley (1996), ISBN 0-471-93001-6
- Multivariate Analysis of Quality: An Introduction, H. Martens and M. Martens, Wiley (2001), ISBN 0-471-97428-5
- Journals
- Journal of Chemometrics
- Chemometrics and Intelligent Laboratory Systems
- Analytical Chemistry
- Analytica Chemica Acta
- Applied Spectroscopy
- Critical Reviews in Analytical Chemistry
- Journal of Process Contro
- Computers in Chemical Engineering
- Technometrics
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